Journal of Sound and Vibration (2000) **233**(5), 929–934
doi:10.1006/jsvi.1999.2705, available online at http://www.idealibrary.com on IDEAL®

A NEW METHOD FOR MEASURING THE BULK MODULUS OF COMPLIANT ACOUSTIC MATERIALS

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(*Received* 17 *September* 1999)

1. INTRODUCTION

In order to characterize the elastic response of any acoustic material, the knowledge of at least two elastic moduli is required. In the literature dealing with the measurements of such parameters [1, 2], it seems that the bulk modulus K has received the least attention, with only a few papers describing experimental procedures for its determination. There are in fact two main methods for measuring the dynamic bulk modulus of compliant materials. The first one is the impedance tube method [3], which is limited to high-frequency measurements, above several kHz. The second one is the method developed by McKinney *et al.* [4] which uses a pressure chamber equipped with two piezoelectric transducers and containing a liquid of known compressibility; the requirement that the sample be uniformly compressed restricts this method to low-frequency measurements. Furthermore, a static pressure has to be applied to avoid the presence of small air bubbles in the liquid.

The present article describes a new method for directly measuring the bulk modulus of compliant acoustic materials at low frequency, without the need for static pressure. It is based on assumption that the strain resulting from uniform compression of the sample can be detected by a laser Doppler interferometer. This paper is divided into three sections: in section 2, the experimental apparatus and the procedure are described; section 3 presents some preliminary data obtained in air at room temperature and under atmospheric pressure on rubber composites; finally, improvements of the present method are discussed in section 4.

2. EXPERIMENTAL PROCEDURE

The laser Doppler interferometer that is used is illustrated in Figure 1. It is a two-sided vibrometer configured for normal (out-of-plane) displacement detection and is a slightly modified version of the apparatus described in detail in reference [5]. The output of each photodiode is processed by a phase-locked loop (PLL) circuit which produces a voltage proportional to the surface velocity of the sample. After integration and calibration, the outputs of each arm of the optical system yield the displacements on opposite sides of the sample.

The experimental arrangement is shown in Figure 2. The sample is a 1·27 cm long cube suspended in air by thin threads that is placed inside a steel tube. A loudspeaker is attached to one end of the tube, and the other end is terminated by a rigid plate equipped with a microphone, next to which the sample is located. The reasons for using the tube are to maximize the pressure amplitude in the vicinity of the sample and to isolate the optical fibers used in the interferometer from the sound field (fibers are very sensitive to pressure fluctuations). The front view of Figure 2 shows light beams passing through glass windows

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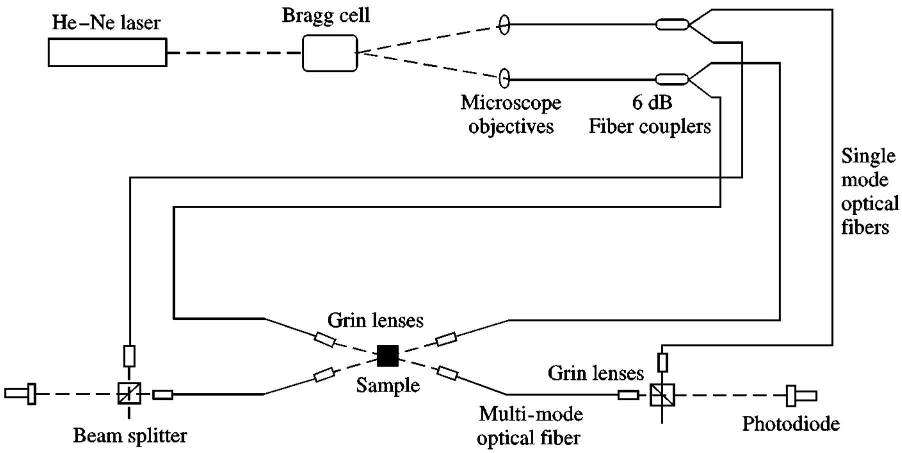

Figure 1. Two-sided optical probe for out-of-plane displacement detection. The solid lines represent optical fibers and the dashed lines are laser beams in air.

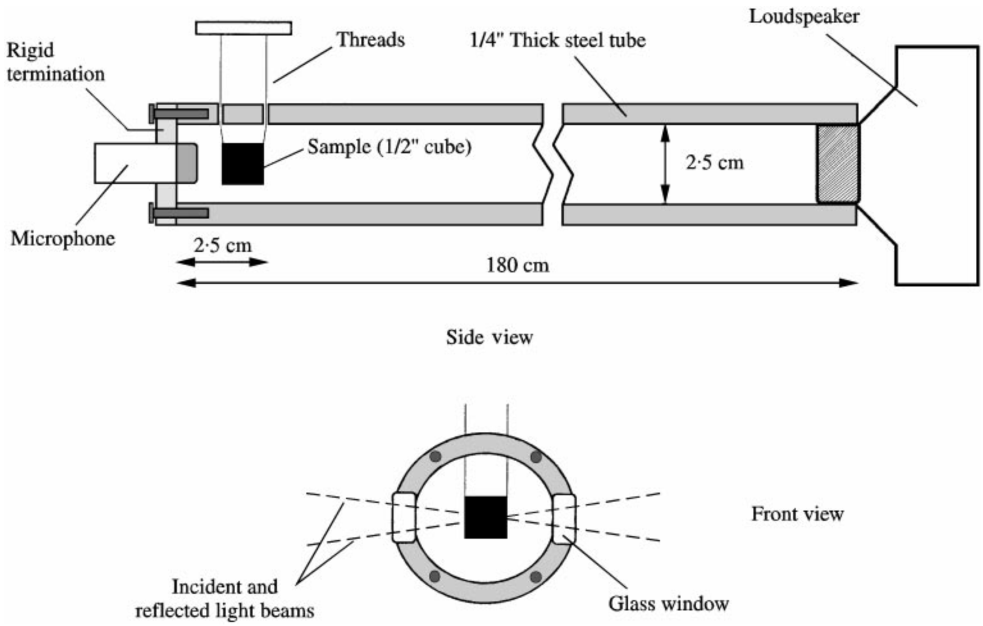

Figure 2. Experimental apparatus for measuring the dynamic bulk modulus, showing the acoustic material sample suspended in air inside the steel tube.

and being reflected on the sample surface (small pieces of plastic reflective tape are glued to the sides of the sample). It also illustrates an inherent difficulty in the procedure: when the pressure fluctuates inside the tube, so does the index of refraction of air. The portions of the optical path located inside the tube are therefore subjected to these fluctuations which are interpreted as displacements by the optical probe. Therefore, the output of the optical probe is the sum of this "apparent displacement" and the actual "sample displacement" resulting from its deformation. To overcome this problem, the measurements have to be calibrated using a metal sample (aluminum cube) of the same dimensions as the acoustic material sample. Aluminum is incompressible with respect to the sensitivity of the system, and thus the probe outputs obtained from the metal cube only reflect the apparent displacement corresponding to this refractive index effect. The latter is subtracted from what is measured on the compliant acoustic material, to obtain its actual deformation.

The experimental procedure is as follows: the loudspeaker is driven with a transient signal made of two cycles at a given frequency, and the pressure P inside the tube is recorded by a sound level meter. The displacement signals from each side of the sample (LS, RS) are then obtained with the interferometer, as well as the corresponding apparent displacements from the aluminum cube (LC, RC). Typical signals are illustrated in Figure 3. All the signals are Fourier-transformed in order to work with their amplitude and phase in the frequency domain. The magnitude of the bulk modulus is therefore given by

$$|K| = \frac{P}{3\Delta l/l}, \tag{1}$$

where $\Delta l = (LS + RS) - (LC + RC)$ and l is the side length of the sample. All the quantities appearing in equation (1) are amplitudes of the signals. One can also compute the loss associated with volume deformation by estimating the phase difference $\Delta\phi$ between corresponding Fourier components of the sample and of the aluminum cube signals. The real and imaginary parts of K (K' and K'', respectively), can then be computed, using

$$|K| = \sqrt{K'^2 + K''^2}, \qquad \tan(\Delta\phi) = K''/K'. \tag{2, 3}$$

The frequency range of the method is approximately 200 Hz to 2 kHz. The lower limit is set by the tube length (it must be long enough so that the original pulse is well separated from the echoes) and by the phase-locked loop circuit (whose signal-to-noise ratio is inversely proportional to frequency). The value of 200 Hz actually corresponds to a 3-m tube and can be improved by using a more sensitive PLL circuit or another way of

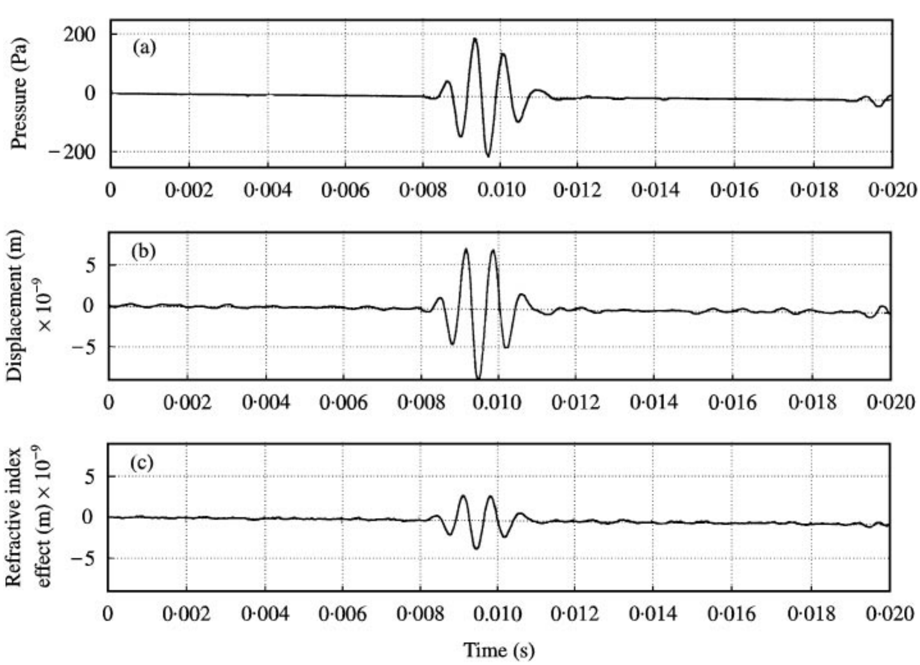

Figure 3. Typical pressure and displacement signals obtained from the system. (a) Transient acoustic signal used to compress the sample. (b) Displacement ($LS + RS$) measured on the sample. (c) "Apparent displacement" ($LC + RC$) measured on the aluminum cube.

demodulating the signal (with a spectrum analyzer, for instance). The higher limit corresponds to the requirement that the sample dimensions be much less than a wavelength in order to avoid significant pressure gradients and to get a uniform compression of the sample. The minimum detectable displacement is set by the PLL and is approximately 2·5 nm at 200 Hz and 0·3 nm at 2 kHz. Finally, the accuracy of the method is mainly limited by the precision of the sound level meter, and is on the order of 6%.

The minimum detectable displacement is one of the factors limiting the maximum value of K which can be measured by the new method. Other factors include the relatively small size of the sample ($\frac{1}{2}$ in cube) and the maximum peak incident pressure (140 dB). With the above values of system parameters, the signal-to-noise ratio is too small to measure the K of homogeneous rubber samples, for which the values of the bulk modulus are an order of magnitude higher than those of rubber–air composites. However, as discussed in section 4, the present system can be redesigned with sufficient increase in signal level for measurement of K both in homogeneous rubbers and in rubber–air composites.

3. BULK MODULUS MEASUREMENT RESULTS

Tables 1–3 present data obtained on three different types of rubber–air composite materials, as a function of frequency. These measurements were made at room temperature and under atmospheric pressure. It is possible to compare them with values previously obtained from wave speed measurements; however, these latter measurements were performed at higher frequencies and one has to extrapolate their results down to the frequency range of interest in this work, thereby introducing a certain degree of uncertainty (there are rubbers for which the value of K is tabulated in this frequency range, but they are much stiffer than the materials studied in this paper, and, in the present configuration, the loudspeaker could not provide enough pressure to get a measurable displacement). However, the estimates of K from the wave speed measurements are considered sufficiently accurate to evaluate the method described in this paper.

The closed-cell neoprene foam was studied by Caille [6], and his measurements indicate that $|K| \cong 4{\cdot}5 \times 10^6$ Pa at 3 kHz (extrapolated from 10 kHz). Two other materials were also investigated. They were chosen because values of K measured using other techniques were available. Both of these materials are rubber–air composites, with $\sim$30% air by volume. However, their exact composition is proprietary and they are denoted as composites 1 and 2 in the tables. Wave speed measurements at 10 kHz yielded $|K| \cong 257 \times 10^6$ Pa for composite 1 (no extrapolation), whereas $|K| \cong 84 \times 10^6$ Pa at 1 kHz for composite 2

TABLE 1

Bulk modulus magnitude of closed-cell neoprene foam ($\rho = 224$ kg/m^3)

Frequency (Hz)	415	720	928	1343
$\vert K\vert$ (10^6 Pa)	2·7	2·9	2·8	3·1

TABLE 2

Bulk modulus magnitude of rubber–air composite 1 ($\rho = 735$ kg/m^3)

Frequency (Hz)	410	708	915	1306
$\vert K\vert$ (10^6 Pa)	106	106	101	124

TABLE 3

Bulk modulus magnitude of rubber–air composite 2 ($\rho = 610$ kg/m^3)

Frequency (Hz)	415	708	915	1318
$\|K\|$ (10^6 Pa)	103	102	93	107

TABLE 4

Loss tangent of closed-cell neoprene foam

Frequency (Hz)	415	720	928	1343
$\Delta\phi$ (deg)	1·5	9·0	11·8	11·2
tan ($\Delta\phi$)	0·03	0·15	0·21	0·20

(extrapolated from 40 kHz). Based on these values, the data obtained by the new method are within an acceptable range, and this establishes the feasibility of the procedure.

Table 4 presents the phase measurements and the resulting loss tangent values of K, obtained on the neoprene foam. The latter is the only material for which such calculations were performed, because the other two exhibit negligible loss. From Caille's work, complex Young's modulus data are available for this material and they indicate that its loss tangent is constant over the (500–3000 Hz) frequency range: $\tan(\Delta\phi) \cong 0{\cdot}44$ (corresponding to $\Delta\phi \cong 24°$). Aside from the lowest-frequency value in Table 4 (which seems to be inaccurate, possibly because of the poor sensitivity of the PLL at this frequency), the data in this table are compatible with Caille's results in the sense that $\tan(\Delta\phi)$ is approximately constant and consistently lower than the Young's modulus loss tangent.

4. FUTURE WORK AND CONCLUDING REMARKS

Measurements of elastic properties of acoustical materials are made to determine the performance of these materials in sound absorption and sound isolation applications. For these applications, any new proposed technique should have the capability to perform measurements under the following conditions: (1) as a function of frequency, (2) as a function of temperature and pressure, (3) both on composites and on their matrix material (for modelling purposes) and (4) on water-loaded samples (for underwater applications). The experimental set-up presented in this paper can be easily modified to address the last three requirements. The proposed improved design is presented in Figure 4, where a similar steel tube is used, but where the sample is attached to the end piece, thereby eliminating the need for the thread holes. In this configuration, the tube can be pressurized and can be filled with water; it can also be placed in a temperature-controlled chamber. A longer sample is used to provide larger displacements, and one of its ends is attached to the back of the tube with rubber cement in order to minimize constraints. The sample elongation is measured at its other end using a single optical probe.

The performance of the system in Figure 4 is estimated for the following conditions: (1) an air-filled, 3 m long tube, operating over the frequency range 200–1000 Hz and (2) a maximum incident sound pressure of 160 dB (re 20 μPa). The lower-frequency limit is set by the condition that the length of the tube should be greater than the wavelength of sound, so that successive echoes can be separated. The upper frequency limit is set by the

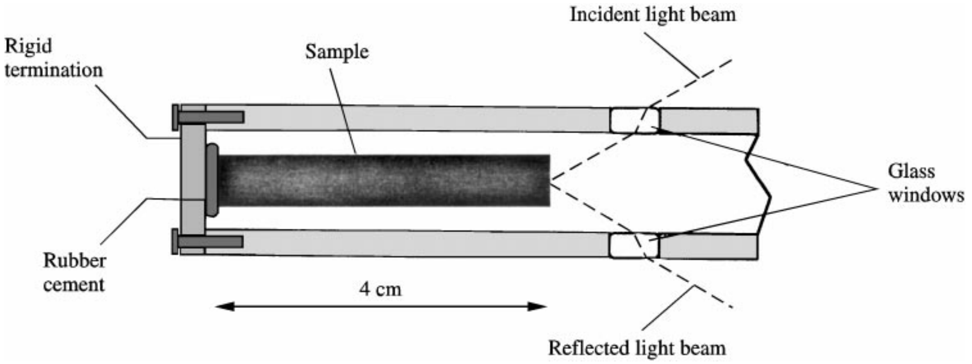

Figure 4. Proposed design to further improve the present method: closed tube with longer sample for measurements as a function of pressure and temperature.

competing conditions that the sample length L not to be too short ($L \geqslant 4$ cm) and that the incident sound pressure be approximately uniform over the length of the sample ($L \leqslant \lambda/8$, where λ is the wavelength of the incident sound). It is estimated that a sound pressure level of 160 dB can be achieved with a magnetic drive piston source combined with a horn concentrator. This level corresponds to an incident peak pressure amplitude of 2×10^3 Pa. Consider a 4 cm long neoprene rubber sample whose adiabatic bulk modulus is $K = 3{\cdot}15 \times 10^9$ Pa at 20°C [3]. Under those conditions, the incident sound will produce a sample elongation $\Delta L = 8{\cdot}5$ nm, which is well above the minimum detectable displacement of the optical probe over the considered frequency range (as presented in section 2). Therefore, the system shown in Figure 4 can be used to measure the complex bulk modulus both for homogeneous rubbers and for rubber–air composites. Furthermore, the steel tube utilized in the method is similar to an impedance tube and can be used both air-filled and water-filled, over a range of temperatures and pressures.

In conclusion, the feasibility of the proposed method was established by obtaining satisfactory preliminary data on compliant composite materials. The advantage of the method is that it provides a direct measurement of the real and imaginary parts of the dynamic bulk modulus at low frequency. This approach can be extended to measurements as a function of pressure and temperature, as well as to measurements with either liquid or gaseous pressure media. Finally, it should be noted that the present technique can be also used for porous materials, and that it will measure the bulk modulus of the porous frame.

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